more involved. We express the interaction (31) in terms of its Fourier transform

$$\exp(-\mu r)=(\mu/\pi^2)\int(k^2+\mu^2)^{-2}\exp(i\mathbf{k}\cdot\mathbf{r})d^3k,$$

which makes the coordinate integrations of the form (C1). There remains the integration over $\mathbf{k}$, of the form

$$\int_0^\infty(k^2+\mu^2)^{-2}(k^2+\gamma^2)^{-7/2}k^2dk,$$

which is elementary, but tedious to evaluate. With suitable substitutions, this leads to Eqs. (49) and (50).

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Three-Pion Decay Modes of Eta and K Mesons and a Possible New Resonance*

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A model which postulates a spin-zero $T=0$ dipion, proposed earlier to explain an apparent enhancement of the three-pion decay mode of the η meson, is applied to obtain detailed predictions concerning the three-pion decays of the η and K mesons. Good agreement is found with all the available data on η and K spectra and branching ratios if the dipion mass and full width are taken as about 400 MeV and 75 to 100 MeV, respectively, thus providing positive evidence for the existence of a two-pion resonance reported by Samios.

I. INTRODUCTION

SINCE its discovery,[1] study of the three-pion decay mode of the eta meson has helped to establish the correctness of the assignments[2,3] spin and parity 0^-, isospin and G parity 0^+, so that its observed pionic decay is a G-forbidden one. Several different theoretical models have been proposed[4–11] to explain and correlate the following features of the three-pion mode: (a) an apparent enhancement of the partial rate relative to $\eta\to 2\gamma$ and also relative to $\eta\to\pi^++\pi^-+\gamma$, (b) the density of the Dalitz–Fabri plot, (c) the ratio $R[=\Gamma_\eta(000)/\Gamma_\eta(+-0)]$ of neutral to charged decays in the 3π mode. Models of η decay have implications for K-meson decays to three pions which permit additional tests to be made of the theory.

The model proposed by the present authors[5] assumed the dominance of a resonant S-wave $T=0$ two-pion component of the three-pion final state to explain qualitatively the enhancement of this partial rate. At the same time, it was noted that a mass near 370 MeV and a width of about 50 MeV for the resonance would give approximate agreement with the Dalitz–Fabri plots then available. A feature which distinguished our model from others subsequently proposed was the ratio R, which was calculated with our theory *in the limit of zero width* as 0.5 or 0.55 if correction is made for the $\pi^\pm$–π^0 mass difference, while the others give $R\approx 1.7$.

We are presenting here the results of a detailed investigation of the consequences of a strongly attractive energy-dependent S-wave two-pion interaction in the $T=0$ state, represented phenomenologically as a dipion "particle" (σ) having a finite width. Good agreement with the Dalitz–Fabri plot for $\eta\to 3\pi$ is obtained for $m_\sigma\approx 400$ MeV, $\Gamma_\sigma=75$ to 100 MeV. For these values, we find $R\approx 1.35$, which can be compared with a recent direct experimental measurement[12,12a] yielding $R=0.83\pm 0.32$. The same model, with the same parameters, applied to the $K\to 3\pi$ decays gives a good fit to the momentum distributions of the unlike pions in both the τ and the τ' modes and gives the branching ratio $\Gamma_K(++-)/\Gamma_K(+00)=3.32$ (for $\Gamma_\sigma=100$ MeV), as compared to the experimental result[13] 3.36 ± 0.28. We also verify that sufficient enhancement of the 3π mode

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1 A. Pevsner, R. Kraemer, M. Nussbaum, C. Richardson, P. Schlein, R. Strand, T. Toohig, M. Block, A. Engler, R. Gessaroli, and C. Meltzer, Phys. Rev. Letters **7**, 421 (1961).
2 P. L. Bastien, J. P. Berge, O. I. Dahl, M. Ferro-Luzzi, D. H. Miller, J. J. Murray, A. H. Rosenfeld, and M. B. Watson, Phys. Rev. Letters **8**, 114 (1962).
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9 Claude Kacser, Phys. Rev. **130**, 355 (1963).
10 S. Okubo and B. Sakita, Phys. Rev. Letters **11**, 50 (1963).
11 B. Barrett and G. Barton (to be published).
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12a C. Bacci, G. Penso, G. Salvini, A. Wattenberg, C. Mencuccini, R. Quenzoli, and V. Silvestrini, Phys. Rev. Letters **11**, 37 (1963). These authors have found $\Gamma_\eta(2\gamma)/[\Gamma_\eta(3\pi^0)+\Gamma_\eta(\pi^0\gamma\gamma)]=0.8\pm 0.25$, where $\Gamma_\eta(\pi^0\gamma\gamma)$ is believed to be small.
13 G. Alexander, S. P. Almeida, and F. S. Crawford, Jr., Phys. Rev. Letters **9**, 69 (1962). The authors whose experimental values have been averaged are quoted in this reference.

persists for reasonable Γ_σ. (Note that $\eta \to \pi^+ + \pi^- + \gamma$ is not enhanced as $\eta \to \sigma + \gamma$ is forbidden by charge-conjugation invariance.)

II. DALITZ PLOTS AND 3π BRANCHING RATIOS FOR η, FOR K^+, AND FOR K_2^0

For concreteness, we shall describe our model in terms of the $\eta \to \pi^+ + \pi^- + \pi^0$ transition which we suppose to take place through the mechanism illustrated in Fig. 1(a). The σ has been assigned a propagator (in the usual notation),

$$D^{-1} = [(p_\eta - p_0)^2 - (m_\sigma - i\Gamma_\sigma/2)^2]^{-1}, \qquad (1)$$

and the vertices $(\eta\sigma\pi)$ and $(\sigma\pi\pi)$ have been assumed to be momentum-independent and characterized by constants αGg and g, respectively. The use of the propagator is justifiable, providing $(p_+ + p_-)^2$ is not too far from m_σ^2 and providing $\Gamma_\sigma/2$ is not too large as compared to m_σ. The energy dependence of the invariant matrix element for $\eta \to \pi^+ + \pi^- + \pi^0$ is then given entirely by D^{-1} and it follows, with a convenient normalization, that the projection of the π^0 kinetic energy T_0 in the Dalitz-Fabri plot has the distribution (in units of the pion mass)

$$F_\eta(T_0) = K(m_\sigma, \Gamma_\sigma) / [(A - T_0)^2 + B^2], \qquad (2)$$

with

$$A = [(m_\eta - \mu)^2 - m_\sigma^2]/2m_\eta, \qquad (2a)$$

$$B = m_\sigma \Gamma_\sigma / 2m_\eta, \qquad (2b)$$

$$K^{-1}(m_\sigma, \Gamma_\sigma) = \int_0^{T_0(\max)} dT_0 [(A - T_0)^2 + B^2]^{-1}. \qquad (2c)$$

The distribution $F_\eta(T_0)$ is plotted in Fig. 2 for several values of m_σ and Γ_σ. Good agreement is obtained with the compiled experimental data[14] if we choose Γ_σ between 75 and 100 MeV and take for m_σ a value between about 390 and 425 MeV. However, it should be noted that in the compilation of Ref. 14 no weight was given to the relative purity of different samples of data and, in fact, the data of different groups, while indicating the same general trend as the compilation, do differ in detail. Thus, values of Γ_σ and m_σ lying slightly outside the ranges indicated above are not excluded.[14a]

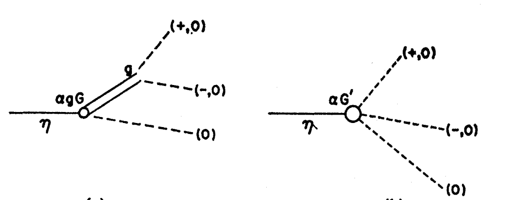

FIG. 1. Feynman diagrams for the decay η (or K) $\to \pi^+ + \pi^- + \pi^0$, all vertices being taken as effectively scalar: (a) assuming the dominance of a σ-dipion intermediate state and (b) assuming a direct decay.

[14] D. Berley, D. Colley, and J. Schultz, Phys. Rev. Letters **10**, 114 (1963).

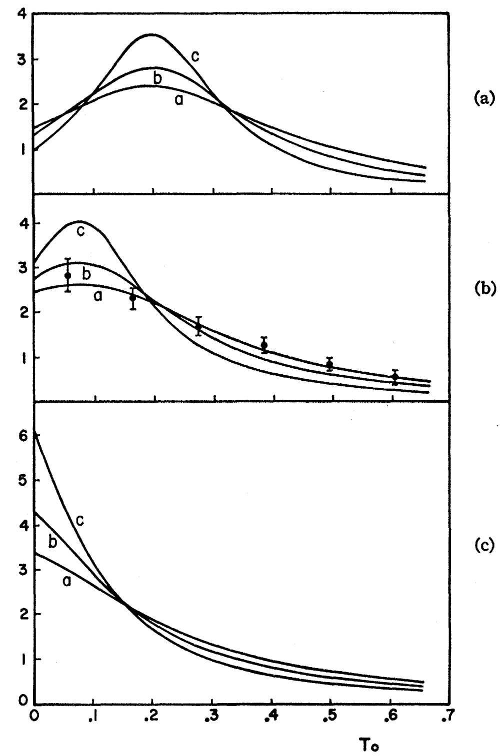

FIG. 2. $F_\eta(T_0)$, the distribution of the π^0 kinetic energy (divided by invariant phase space) in the decay $\eta \to \pi^+ + \pi^- + \pi^0$. Figs. 2(a), 2(b), and 2(c) are for $m_\sigma = 375$, 400, and 425 MeV respectively, while curves labeled a, b, and c are for $\Gamma_\sigma = 100$, 75, and 50 MeV, respectively. The experimental points are taken from Ref. 14.

The rates $\Gamma_\eta(+-0)$ and $\Gamma_\eta(000)$ are calculated [see Fig. 1(a)] as

$$\Gamma_\eta(+-0) = \alpha^2 G^2 g^4 / (4\pi)^3 (4m_\eta^3) I, \qquad (3)$$

$$\Gamma_\eta(000) = [\alpha^2 G^2 g^4 / (4\pi)^3 (4m_\eta^3)] (\beta/3!) (3I + 3J), \qquad (4)$$

where

$$I = \int_\mu^{\omega_{\max}} \frac{d\omega\, \varphi(\omega)}{(A + \mu - \omega)^2 + B^2}, \qquad (5)$$

$$J = \int_\mu^{\omega_{\max}} \frac{d\omega (\omega - A - \mu)}{(A + \mu - \omega)^2 + B^2} \ln \frac{(h + \varphi)^2 + 4B^2}{(h - \varphi)^2 + 4B^2} + \int_\mu^{\omega_{\max}} \frac{2B d\omega}{(A + \mu - \omega)^2 + B^2} \tan^{-1} \frac{4B\varphi}{h^2 - \varphi^2 + 4B^2}, \qquad (6)$$

[14a] *Note added in proof.* Fitting our formula (2) to 97 eta decays, $\eta \to \pi^+ + \pi^- + \pi^0$, having negligible background and contamination, a Berkeley-Duke group has determined as best-fit parameters $m_\sigma = 381 \pm 5$ MeV, $\Gamma_\sigma = 48 \pm 8$ MeV and, using our Eqs. (5), (6), and (8) with these parameters, have predicted $R = 1.02 \pm 0.07$. We wish to thank Professor Frank S. Crawford, Jr., for sending us a preprint of this work. However, while this apparent agreement with our theory is gratifying, the resonance parameters so determined would give too large a slope to the τ-meson spectrum (Fig. 4

where μ is the (average) pion mass and

$$\omega_{\max}=(m_\eta^2-3\mu^2)/2m_\eta, \tag{7a}$$

$$\varphi=(m_\eta^2-2\omega m_\eta-3\mu^2)^{1/2}(\omega^2-\mu^2)^{1/2} \times(m_\eta^2-2\omega m_\eta+\mu^2)^{-1/2}, \tag{7b}$$

$$h=m_\eta-\omega-2A-2\mu, \tag{7c}$$

and $\beta\approx 1.1$ is a correction factor to take account of the $\pi^\pm-\pi^0$ mass difference. The arctangent is required to lie between $-\pi$ and π. (The expression J is twice the real part of the interference term which arises from the exchange of two π^0's.) The ratio

$$R=\Gamma_\eta(000)/\Gamma_\eta(+-0)=(\beta/2)[1+(J/I)] \tag{8}$$

evidently approaches the value 0.55 as the interference term J vanishes, in the limit $\Gamma_\sigma\rightarrow 0$. For finite Γ_σ, R ranges from this value up to 1.73, which is attained for a matrix element having no effective energy variation. Table I contains values of R for m_σ and Γ_σ consistent with the experimental Dalitz plot for η decay.

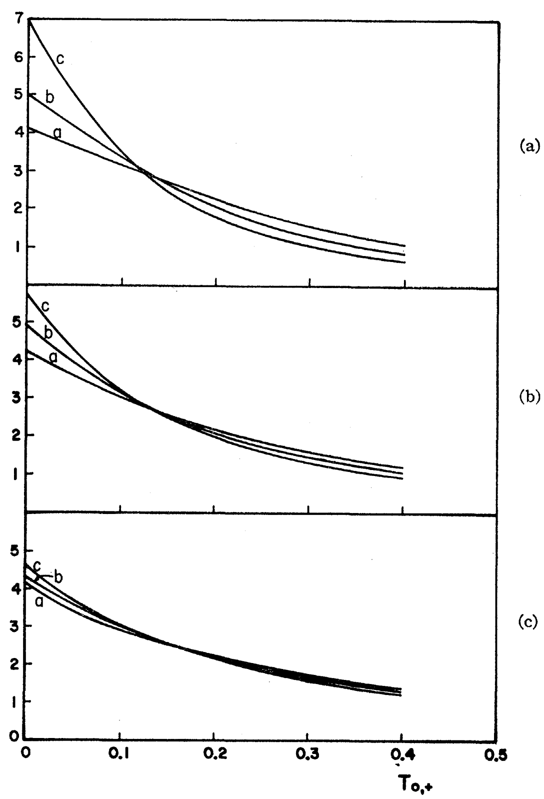

FIG. 3. $F_{K^{+,0}}(T_{\pi^{+,0}})$, the distribution of the π^+ kinetic energy T_{π^+} in the decay $K^+\rightarrow\pi^++\pi^0+\pi^0$ and of the π^0 kinetic energy T_{π^0} in the decay $K_2^0\rightarrow\pi^++\pi^-+\pi^0$ (divided by invariant phase space). Figs. 3(a), 3(b), and 3(c) are for $m_\sigma=375$, 400, and 425, respectively, while curves labeled a, b, and c are for $\Gamma_\sigma=100$, 75, and 50 MeV, respectively. For comparison with experiment, see text.

below) which suggests that somewhat larger values of the mass and width, such as $m_\sigma=390$ MeV and $\Gamma_\sigma=75$ MeV would be more compatible with the data, considered as a whole.

We now apply our model to the various $K\rightarrow 3\pi$ transitions, assuming that the σ dipion again plays an essential role in determining the structure of the final state. This implies, of course, that the final state is pure $I=1$, as in the η case, which is consistent with the rule $|\Delta I|=\frac{1}{2}$ for nonleptonic weak decays, but does not exclude $|\Delta I|=\frac{3}{2}$. In fact, a small $|\Delta I|=\frac{3}{2}$ admixture may be required to explain the ratio of K^+ to K_2^0 three-pion rates. Aside from this ratio, our other results, which are all in good agreement with experiment, are independent of any assumption concerning this admixture.

For the decay $K_2^0(+-0)$, the π^0 kinetic-energy distribution and for $K^+(+00)$ the π^+ kinetic-energy distribution (square of the invariant matrix element) are given by $F_{K^{+,0}}(T_{\pi^{+,0}})$, which is the same as $F_\eta(T_0)$ of Eq. (2) with m_η replaced by the appropriate K-meson mass. Figure 3 gives $F_{K^0}(T_{\pi^0})$ for various choices of m_σ and Γ_σ. $F_{K^+}(T_{\pi^+})$ is essentially given by the same set of curves, except that the maximum value of T_{π^+} is 0.381 μ while for T_{π^0} it is 0.399 μ_0, where μ and μ_0 are the masses of π^+ and π^0, respectively. The experimental statistics for the two-decay modes are not sufficient to permit a detailed comparison to be made with our predictions. Luers *et al.*[15] have reported 58 $K_2^0(+-0)$ events while Bøggild *et al.*[16] have presented a compilation of 119 $K^+(+00)$ events. Luers *et al.* have fitted their $F(T_0)$ roughly with a straight-line $F(T_0)=1+a(T_0/\mu_0)$ with $a=-2.31\pm 0.88$. Our curves with m_σ between 375 and 400 MeV and Γ_σ between 75 and 100 MeV agree well with this approximate fit to the experimental data. For comparison, we have fitted the τ' data of Bøggild with an analogous straight-line $F(T_{\pi^+})=1+a'(T_+/\mu)$ obtaining $a'=-2.2\pm 0.8$, which again agrees with our curves for the parameter tange given above.

TABLE I. Ratio R of neutral to charged three-pion decay of the η meson for various dipion masses m_σ and widths Γ_σ.

m_σ	Γ_σ	R
375 MeV	100 MeV	1.36
375	75	1.21
400	100	1.40
400	75	1.32
425	100	1.49

A more severe test of the theory is provided by the τ decay $K^+(++-)$, for which much better data exist. The projection of the Dalitz-Fabri plot on the π^- energy axis is given by a somewhat more complicated expression in this case, as there are contributions from two interfering diagrams of the type of Fig. 1(a). It is

[15] D. Luers, I. S. Mittra, W. J. Willis, and S. S. Yamamota, Phys. Rev. Letters 7, 255, 361 (1961).

[16] J. K. Bøggild, K. H. Hansen, J. E. Hooper, M. Schaerf, and P. K. Aditya, Nuovo Cimento 19, 621 (1961).

normalized to be unity at half the maximum of T_{π^-},

$$F_{K^+}(T_{\pi^-})=0.133\left[h^{-1}\ln\frac{(h+\varphi)^2+4B^2}{(h-\varphi)^2+4B^2}+B^{-1}\tan^{-1}\frac{4B\varphi}{h^2-\varphi^2+4B^2}\right]. \quad (9)$$

The quantities B, φ, and h are those given previously, but with substitution of the appropriate masses. This function is given in Fig. 4 for $m_\sigma=400$ and $\Gamma_\sigma=100$ MeV. The experimental points have been plotted with the same normalization and are taken directly from the compilation of Ferro-Luzzi *et al.*[17] The agreement, evidently, is excellent. Because, for this decay unlike the η case, the σ-dipion peak lies outside the kinematically allowed energy region, and also because of interference effects, F_{K^+} is less sensitive than the other distributions presented to small variations in the parameters m_σ and Γ_σ. For this reason, we have given only a single curve for this distribution.

The partial widths for the various $K\rightarrow 3\pi$ decays are calculated to be

$$\Gamma_{K^0}(+-0)=C_0I'(+-0); \quad (10a)$$

$$\Gamma_{K^0}(000)=(C_0/3!)(3I'(000)+3J'(000)); \quad (10b)$$

$$\Gamma_{K^+}(++-)=(C_+/2!)(2I'(++-)+J'(++-)); \quad (10c)$$

$$\Gamma_{K^+}(+00)=(C_+/2!)I'(+00). \quad (10d)$$

I' and J' are obtained from expressions like (5) and (6) with substitution of the appropriate masses. Because of the limited phase space available for the K-meson decays, we have used a more accurate expression in which we have not neglected the pion-mass differences; therefore, the correction factor β does not appear in Eq. (10). For our usual choice of parameters $m_\sigma=400$, $\Gamma_\sigma=100$ MeV, we predict the branching ratios

$$\Gamma_{K^+}(+00)/\Gamma_{K^+}(++-)=0.302 \quad (11)$$

and

$$\Gamma_{K^0}(000)/\Gamma_{K^0}(+-0)=1.66. \quad (12)$$

The values of the constants C_0, C_+ involve assumptions concerning the weak interaction; for example, the prediction of the $|\Delta I|=\frac{1}{2}$ rule is that $m_{K^0}{}^3C_0=m_{K^+}{}^3C_+$. For this case, we get

$$\Gamma_{K^0}(+-0)/\Gamma_{K^+}(+00)=1.95I'(+-0)/I'(+00)=2.22. \quad (|\Delta I|=\tfrac{1}{2}) \quad (13)$$

The experimental situation is the following: to be compared with Eq. (11) is the value 0.298 ± 0.025, which is an average of several experiments.[13] Another test which is independent of the rule $|\Delta I|=\frac{1}{2}$ is furnished by Eq. (12); however, the only existing published experimental result,[18] while not inconsistent with Eq. (12), is too uncertain to be claimed as a significant test of the theory. By contrast, the ratio of Eq. (13) is almost independent of the parameters which determine our model and is sensitive to the validity of the $|\Delta I|=\frac{1}{2}$ rule. The best experimental value[19] to be compared with Eq. (13) is reported to be in good agreement with this rule and, hence, with the theoretical value quoted in Eq. (13).

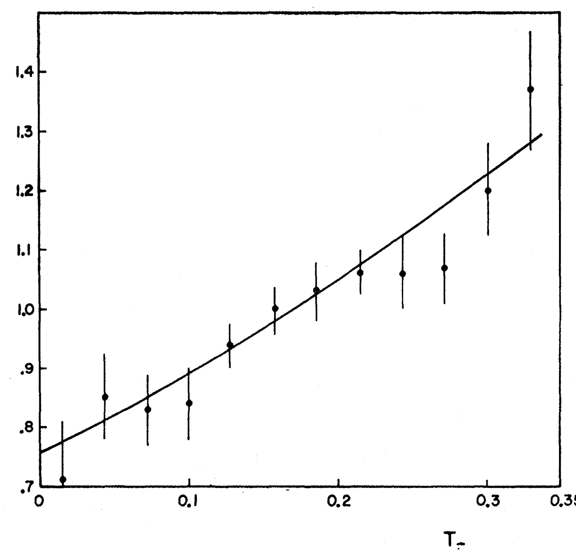

FIG. 4. $F_{K^+}(T_{\pi^-})$, the distribution of the π^- kinetic energy (divided by invariant phase space) in the decay $K^+\rightarrow\pi^++\pi^++\pi^-$. The theoretical curve is for $m_\sigma=400$, $\Gamma_\sigma=100$ MeV and is normalized to unity at half the maximum of T_{π^-}. Experimental points, with the same normalization, are taken from Ref. 17.

III. DECAY RATE OF THE η MESON

We now turn to the task of estimating an absolute three-pion decay width for the η meson. As we have remarked earlier, our model was originally introduced for the purpose of explaining an apparent large enhancement in the mode $\eta\rightarrow 3\pi$, using as a comparison mode, for example, $\eta\rightarrow 2\gamma$. For a σ meson of narrow width appearing close to its mass shell, it is intuitively clear that this enhancement will be close to the ratio of two-body to three-body phase space. We have seen above that with a more realistic width of 75 to 100 MeV all the details of η and K decay into three pions are

[17] M. Ferro-Luzzi, D. H. Miller, J. J. Murray, A. H. Rosenfeld, and R. D. Tripp, Nuovo Cimento 22, 1087 (1961). We have averaged adjacent points to obtain half the number of intervals used by the above authors.

[18] M. H. Anikina, D. M. Kotliarevsky, A. A. Koslov, M. S. Dzurarleva, and S. M. Mandzavidse, in *Proceedings of 1962 International Conference on High-Energy Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962), p. 452. Professor R. H. Dalitz reported at the International Conference on Weak Interactions [Brookhaven National Laboratory, September 1963 (to be published)], based in part on a revision of the above work, an experimental average for the ratio of Eq. (12) as 1.62 ± 0.6.

[19] This value, based on 17 $K_2{}^0$ events has been obtained by a Wisconsin–Berkeley collaboration (to be published). We wish to thank Professor W. F. Fry for this information.

TABLE II. Absolute rate $\Gamma_\eta(+-0)$ of $\eta \to \pi^+ + \pi^- + \pi^0$.

m_σ	Γ_σ	$\Gamma_\eta(+-0)$
375 MeV	100 MeV	350 G^2 eV
400	100	265 G^2
400	50	155 G^2

well-explained. We now wish to verify that for the η case the enhancement of the three-pion mode persists.

Our approach to this problem may be seen by a comparison of the diagrams in Fig. 1. The first [Fig. 1(a)] corresponds to the calculational method that we have used in the earlier sections of this work. The second diagram is meant to indicate a hypothetical decay in which no resonance appears. The η vertices in the graphs designate vertex structures[20] in which a virtual photon occurs, hence the appearance of the fine structure constant α.

The constant g^2 is related to the width of the σ particle Γ_σ by $(\frac{2}{3})\Gamma_\sigma = g^2\rho_2$, where ρ_2 is the phase space for the decay $\sigma \to \pi^+ + \pi^-$. We thus obtain

$$g^2 = \Gamma_\sigma (32\pi m_\sigma^2)/3(m_\sigma^2 - 4\mu^2)^{1/2}. \tag{14}$$

For $m_\sigma = 400$ MeV, $\Gamma_\sigma = 100$ MeV, this gives $(g^2/4\pi m_\sigma^2) = 0.9$. The absolute three-pion rate of η decay is calculated from Eq. (3) and is given in Table II. For the reasonable value $G^2 \approx 1$, which makes the $(\eta\sigma\pi)$ coupling equal to α times the $(\sigma\pi\pi)$ coupling, and for the parameters fitted to the spectra and branching ratios in Sec. II we obtain $\Gamma_\eta(+-0)$ of the order of 200 eV. This can be compared, for example, with $\Gamma_\eta(2\gamma)$, which is theoretically estimated[5,10] as about 160 eV, and with the experimental fact that the modes occur with about equal frequency.

On the other hand, a straightforward calculation of the process in Fig. 1(b) gives the result

$$\begin{aligned} \Gamma_\eta'(+-0) &= \alpha^2 G'^2 [0.25/(4\pi)^3 m_\eta] \\ &= 2.2 G'^2 \text{ eV}. \end{aligned}$$

The constant G'^2 is *not* to be fitted from any experiment, since in our view Fig. 1(b) represents a purely hypothetical decay. But we see that to obtain experimental agreement we would have had to assume $G'^2 \approx 100 G^2$ and this factor crudely measures the enhancement of the decay rate by the σ dipion.

To study the dependence of the $\eta \to 3\pi$ partial width on the strength of the $\sigma \to 2\pi$ transition, we note that this dependence is actually given by the quantity $g^2 I$. This follows from the requirement that in the limit $g^2 \to 0$ the process $\eta \to 3\pi$ becomes the process $\eta \to \sigma + \pi$ with stable σ. That is, as shown in the Appendix, in order to have

$$\lim_{g^2 \to 0} \Gamma_\eta(+-0) = \tfrac{2}{3}\Gamma_\eta(\sigma\pi^0)$$

it is necessary that

$$G_{\eta\sigma\pi}^2 = \lim_{g^2 \to 0} g^2 G^2,$$

where the $(\eta\sigma\pi)$ vertex is given by $\alpha G_{\eta\sigma\pi}$. This suggests that it is reasonable to assume that gG is constant for not too large Γ_σ, from which the $g^2 I$ dependence follows. Thus, for $m_\sigma = 375$ and $\Gamma_\sigma = 50$, 75, 100 MeV, $g^2 I$ is in the ratio 3.9:3.4:2.9, while for $\Gamma_\sigma = 50$, $m_\sigma = 375$, 400, 450 MeV, the $g^2 I$ ratios are 7.9:4.8:3.4. This behavior is in accord with the qualitative picture of enhancement.

IV. IS THERE A σ RESONANCE?

Our work has been based on the assumption that there exists a strong energy-dependent attractive interaction in the S-wave $T=0$ two-pion system, peaking at an invariant mass of about 400 MeV and with a width of about 75 to 100 MeV. Predictions deduced from this model agree consistently, and indeed remarkably well, with all the available information on K and η decay. We believe, therefore, that an interaction of this nature exists, and we believe that it is probably a resonance. As a rather large number of experiments has been performed in which at least two pions appear among the outgoing particles, this would seem to be susceptible of straightforward test. In fact, the subject is imbued with controversy.

Thus, Samios *et al.*[21] have compared like and unlike charged-pion pairs from 4.7 BeV/c π^- on protons. They observe the effective-mass distributions to be markedly different for the like and unlike pairs and infer the existence of $T=0$ or 1 resonances at 395 ± 10 and 520 ± 20 MeV. Dipion effects at effective mass about 400 MeV had been reported earlier[22,23] and have been reported subsequently.[24] These include much more copius production in those two-pion channels in which the mass-spectrum anomaly appears. However, no such effect appears to be present in $\pi^+ p$ experiments[25] at

[20] In this sense, our model is not orthogonal to those proposed by other authors, especially those in which the electromagnetic (or, in the K-meson case, the weak) decay process is assumed to proceed to a single-pion intermediate state, followed by a strong interaction of the form $\lambda\varphi^4$, since nothing definite has been assumed about the structure of the $(\eta\sigma\pi)$ vertex. In fact, if the σ resonance dominates S-wave pion-pion scattering in the low-energy experiments from which λ is fitted, we would expect such a theory to provide the necessary enhancement of the $\eta \to 3\pi$ decay rate (not, however, its explanation); but it will not yield the spectra without further assumptions.

[21] N. P. Samios, A. H. Bachman, R. M. Lea, T. E. Kalogeropoulos, and W. D. Shephard, Phys. Rev. Letters **9**, 139 (1962). Note that $T=1$ is excluded by the absence of these resonances in the ω Dalitz plot (Ref. 25).

[22] J. Kirz, J. Schwartz, and R. D. Tripp, Bull. Am. Phys. Soc. **7**, 48 (1962).

[23] C. Richardson, R. Kraemer, M. Meer, M. Nussbaum, A. Pevsner, R. Strand, T. Toohig, and M. Block, in *Proceedings of the 1962 International Conference on High-Energy Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962).

[24] J. Kirz, J. Schwartz, and R. D. Tripp, Phys. Rev. **130**, 2481 (1963) and earlier references quoted in this work.

[25] C. Alff, D. Berley, D. Colley, N. Gelfand, V. Nauenberg, D. Miller, C. Schultz, J. Steinberger, T. H. Tan, H. Brugger, P. Kramer, and R. Plano, Phys. Rev. Letters **9**, 322, 325 (1962).

2.34 and 2.90 BeV/c. It is worth remarking that it may be difficult to observe this resonance when strong competition is present either from the ρ meson or, and especially, from the nucleon isobar. Olsson and Yodh[26] have made an extensive analysis of single-pion production in pion-nucleon collisions between 350 and 1000 MeV using an improved (3,3) isobar model and conclude that the model fails *only* in the reaction $\pi^-+p\rightarrow\pi^-+\pi^++n$, indicating the existence of "some two-pion interaction in isotopic spin-zero state which modifies the simple isobar model." In view of these isobar complications, it would appear desirable to search for this resonance in, for example, $\bar{K}p\rightarrow\sum\pi\pi$.[27]

We shall not speculate about the relationship of the σ dipion to the explanation of the so-called "*ABC* anomaly,"[28] except to note that these are presumably different effects, although they appear in the same dipion channel, since the latter effect occurs at threshold. However, if the σ resonance exists it must surely be taken into account in any theoretical treatment of *ABC*, as well as in a long list of other problems including all baryon-force problems.

If a 400-MeV 0^+ resonance having $T=0$ exists, the question of where it would fit in the unitary-symmetry scheme arises. Of course, it could be a unitary singlet; but an unassigned $K\pi$ resonance exists at 725 MeV which could have spin and parity 0^+ and which could be associated with the 400-MeV 0^+ resonance as part of a scalar unitary octet. Continuing with this speculation, we would infer (using the Gell-Mann/Okubo mass formula) a scalar isotropic triplet of mass 1275 MeV, which should appear predominantly as a π–η resonance, since strong decay into fewer than five pions would be forbidden.

In conclusion, we should like to emphasize that the η-meson decay into three pions (especially the low kinetic-energy end of the π^0 spectrum) is very sensitive to the mass and width of the proposed σ resonance. The reason for this is that the Q value for $\eta\rightarrow\sigma+\pi^0$ is very small for the relevant mass of σ.

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APPENDIX

We wish to study the process shown in Fig. 1(a) in the limit $g\rightarrow 0$ to obtain the dependence of the enhancement of the $\eta\rightarrow 3\pi$ mode on the width Γ_σ of the σ dipion as discussed in Sec. III. We separate the factor g^2I in Eq. (3) and write it

$$g^2I=\frac{g^2}{B}\int_\mu^{\omega_{\max}}\frac{\varphi(\omega)d\omega B}{(A+\mu-\omega)^2+B^2}. \tag{A1}$$

Noting from Eq. (14) and Eq. (2b) that

$$g^2/B=(64\pi/3)m_\eta m_\sigma(m_\sigma^2-4\mu^2)^{-1/2} \tag{A2}$$

and that

$$\lim_{g^2\rightarrow 0}\frac{B}{(A+\mu-\omega)^2+B^2}=\pi\delta(A+\mu-\omega), \tag{A3}$$

so that for the dipion masses that we have been considering we obtain

$$\lim_{g^2\rightarrow 0}g^2I=(64\pi^2/3)m_\eta m_\sigma(m_\sigma^2-4\mu^2)^{-1/2}\varphi(A+\mu), \tag{A4}$$

where

$$\varphi(A+\mu)=(m_\sigma^2-4\mu^2)^{1/2}\times[(m_\eta^2+\mu^2-m_\sigma^2)^2-4m_\eta^2\mu^2]^{1/2}/2m_\sigma m_\eta. \tag{A5}$$

However, for an essentially stable dipion, we would have

$$\Gamma_\eta(+-0)=\tfrac{2}{3}\Gamma_\eta(\sigma\pi^0), \tag{A6}$$

while for the two-body decay $\eta\rightarrow\sigma+\pi^0$

$$\Gamma_\eta(\sigma\pi^0)=(\alpha^2G_{\eta\sigma\pi}^2/16\pi m_\eta^3)\times[(m_\eta^2-m_\sigma^2-\mu^2)^2-4\mu^2m_\sigma^2]^{1/2}. \tag{A7}$$

Here, we have let the $\eta\rightarrow\sigma+\pi^0$ vertex be characterized by the coupling constant $\alpha G_{\eta\sigma\pi}$. Referring again to Eq. (3) we obtain, using Eqs. (A4), (A5), and (A7),

$$\lim_{g^2\rightarrow 0}\Gamma_\eta(+-0)=\left(\lim_{g^2\rightarrow 0}g^2G^2\right)G_{\eta\sigma\pi}^{-2}\left(\frac{2}{3}\right)\Gamma_\eta(\sigma\pi^0). \tag{A8}$$

Thus, we can satisfy Eq. (A6), providing we let

$$\lim_{g^2\rightarrow 0}g^2G^2=G_{\eta\sigma\pi}^2. \tag{A9}$$

We inferred in Sec. III, from this relation, that the dependence of the enhancement on the width is given by g^2I.